Reduction in work efficiency due to the number of people

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Introduction

Our Question

How many people can work optimally?

Background

What is the best way to promote?

Peter Principle/Pluchino et al.

Won the Ig Nobel Prize

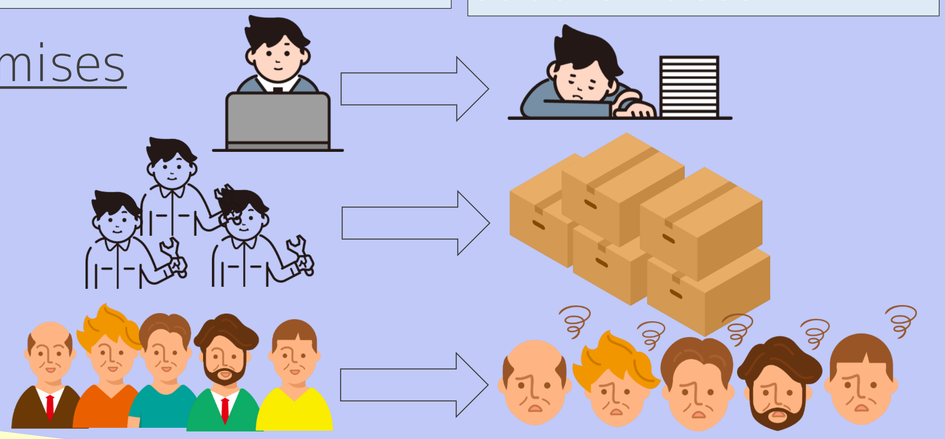
What is the relationship between health and work?

How should we switch tasks?

What is the best balance between work and breaks?

How to manage your subordinates?

Premises



Previous research

Concept of Satoi's model

Satoi's Model

$$\begin{cases} \varphi = \int_0^{\tau} \boldsymbol{a}\left(\boldsymbol{e}^{-ct^2} - 1 + \boldsymbol{\alpha}\right)\boldsymbol{dt} + \int_{\tau}^{T} \boldsymbol{b}\left(\boldsymbol{e}^{-s(t-\tau)^2} - 1 + \boldsymbol{\beta}\right)\boldsymbol{dt} \\ \beta = \boldsymbol{e}^{-\tau^2} - 1 + \alpha \end{cases}$$

Premises

1. Performing the same task for some hours results in the decline of work efficiency.
2. Switching the task to another at a certain time results in resetting the motivation.

Scenario

Task A is performed until a certain time (t). After this, Task B is performed instead of Task A.

Reduction of work efficiency

Task A: $a(e^{-ct^2} - 1)$

Task B: $b(e^{-st^2} - 1)$

Work efficiency at the start

1. Task A: α
2. Task B at a certain time(t): $\beta = e^{-c\tau^2} - 1 + \alpha$

⊛$a = c = 1$

(To see the difference between Task A and Task B.)

$$\begin{cases} \varphi = \int_0^{\tau} \left(\boldsymbol{e}^{-t^2} - 1 + \boldsymbol{\alpha}\right)\boldsymbol{dt} + \int_{\tau}^{T} \boldsymbol{b}\left(\boldsymbol{e}^{-s(t-\tau)^2} - 1 + \boldsymbol{\beta}\right)\boldsymbol{dt} \\ \beta = \boldsymbol{e}^{-\tau^2} - 1 + \alpha \end{cases}$$

Simplification of Satoi's model

Premises

1. Difficulty of the task: Task A = Task B
2. Working hour of each task: Task A = Task B
3. Individual capacity: Everyone is equal.

$$\begin{cases} \varphi = \int_0^{\tau} \left(e^{-t^2}\right)dt + \int_{\tau}^{T} \left(e^{-t^2} - 1 + \beta\right)dt \\ \beta = e^{-\tau^2} \end{cases}$$

Result

The line of Figure 2 is more gradual than that of Figure 1.This simplified model has given the same result as Satoi's model in terms of the decline of work efficiency.

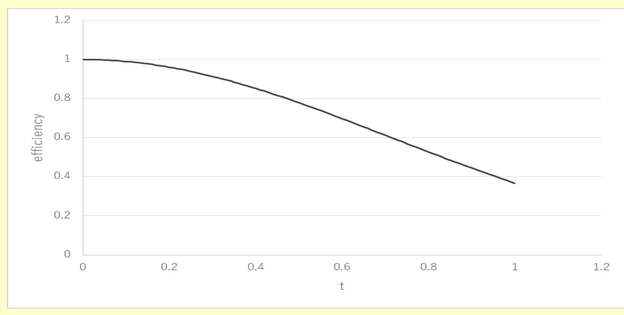


Figure 1: Performing only Task A.

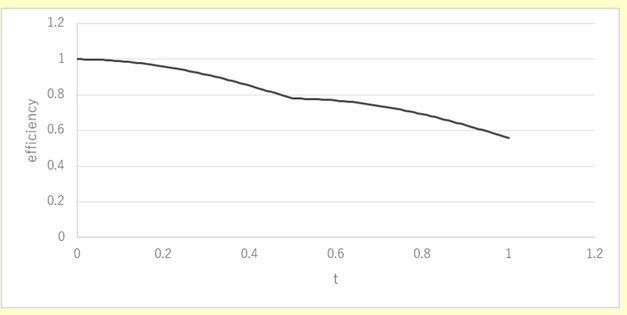


Figure 2: Performing Task A followed by Task B.

A model that takes into account headcount effects using the simplification of Satoi's model

Model overview

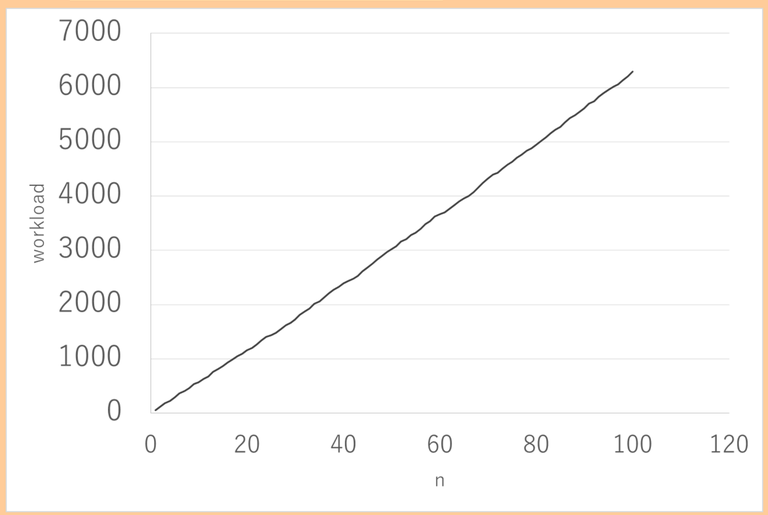


Figure 3
$k = 0$

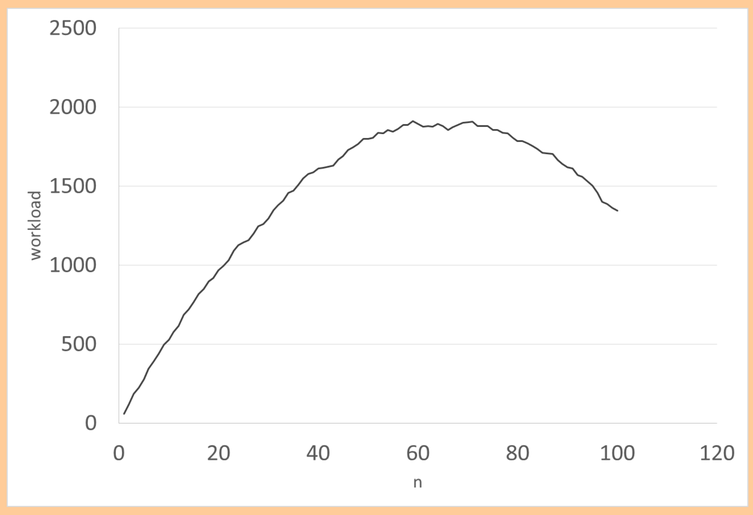


Figure 4
$k = 1$

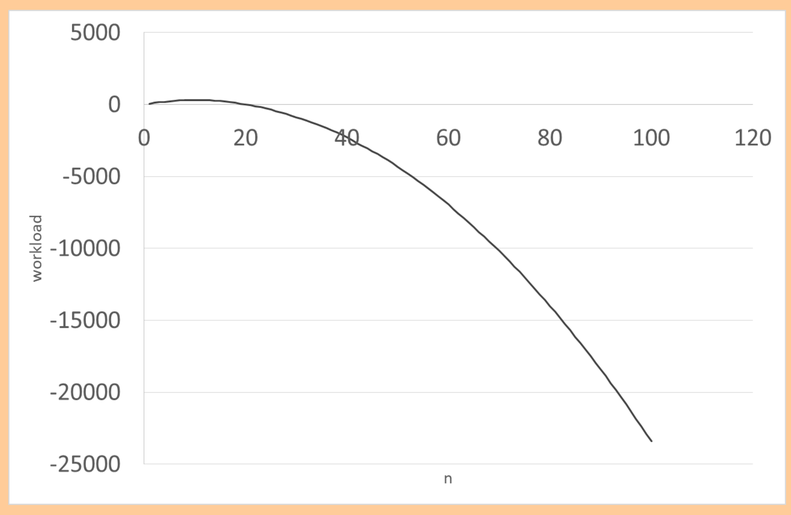


Figure 5
$k = 5$

Model: $\begin{cases} W' = \sum_{i=1}^{n} \left[r_i \left\{\int_0^{\tau} \left(e^{-t^2}\right)dt + \int_{\tau}^{T} \left(e^{-t^2} - 1 + \beta\right)dt\right\}\right] \\ W = W' - k * {}_nC_2 \end{cases}$

We calculated the workload per person from a simplified version of Satoi's model. We used a random number to represent the difference in ability, and added up the number of people to get the total workload as W'. The number of communications was multiplied by the factor k to represent the reduction in workload due to communication.

Considerations

Model considerations

The k value represents decreasing workload when you communicate with one person.

If the k value is 0: the higher the number of people is, the larger the workload is.

If the k value is over 5: the higher the number of people is, the smaller the workload is. This creates problems for work efficiency. Therefore, when work efficiency is less than 90%, fewer people are better.

When the k value is over 0 to under 5: the optimal number of people can be found depending on the k value.

Future outlook

The optimization of work has already been mathematically modeled from various angles. From now on, it is important to create a mathematical model which is related to general work efficiency by integrating the existing models. Also, owing to the fact that most of the models are ones that are based on simple tasks, further research based on long-term and more difficult tasks should be considered for developing a more successful organization.

References

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