Polya's Pot

Nagasaki Prefectural Omura High School

W202010

1. Research Motivation

When we learned about the Polya's pot, we wanted to research new solutions.

2. Purpose

Our purpose is to solve Polya's Pot in a different way.

3. Problem

There are "a" red balls and "b" white balls in a pot.

Trial

①. You pull one ball from the pot at random.

②. You put it back and add one ball whose color matches the color of the ball you pulled (Trial 2).

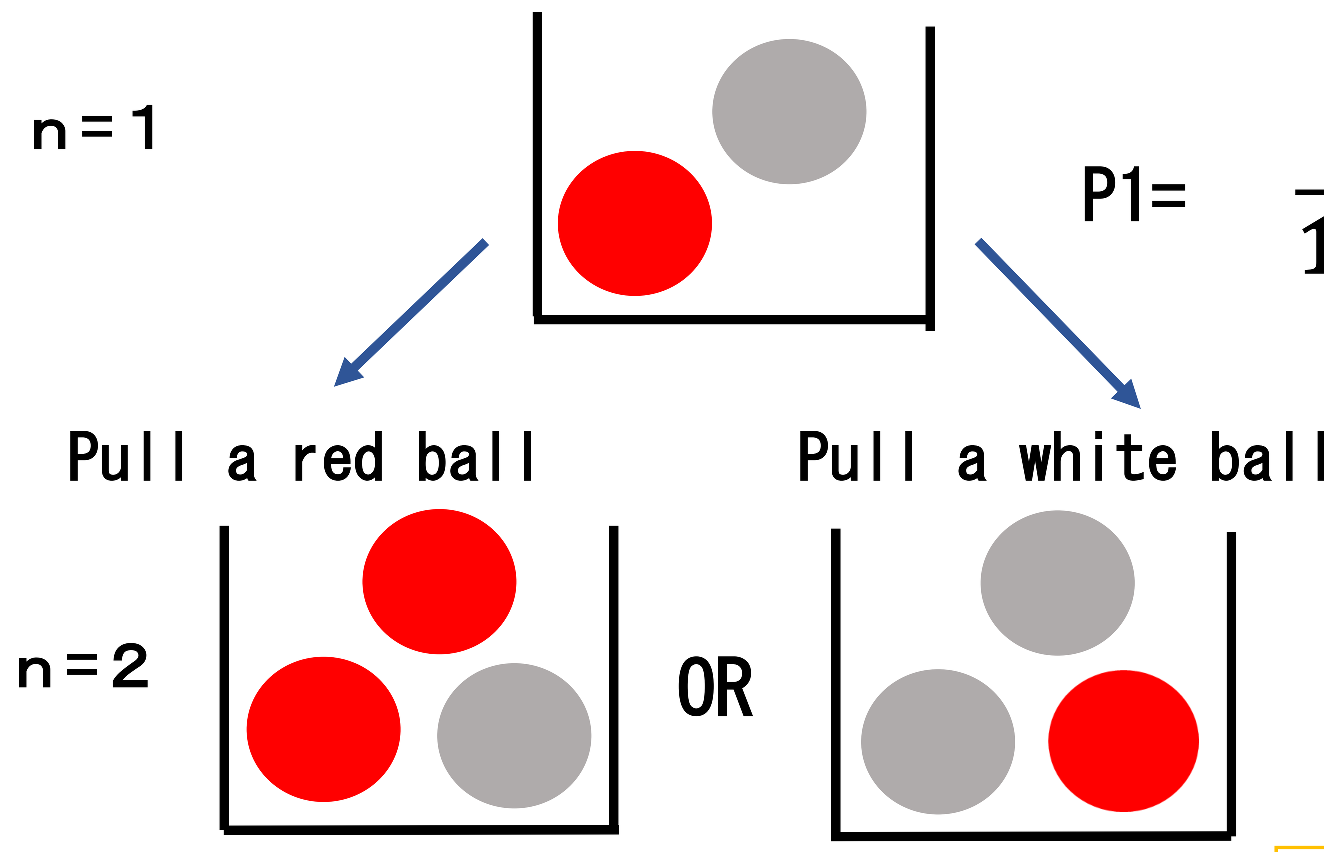
Solution of Generation
・Mathematical Induction
・Recurrence Formula

↓

We use a method that uses combination.

Pn: Probability that you will pull a red ball on the nth iteration.

・**For Example**: The pot has a red ball and a white ball.



$$P1=\frac{1}{1+1}=\frac{1}{2}$$

$$P2=\frac{1}{2}\times\frac{2}{3}+\frac{1}{2}\times\frac{1}{3}=\frac{1}{2}$$

P1= P2

Probability that you will pull a red ball by the Nth iteration.

$$\frac{a}{a+b}$$ ← Doesn't depend on "n"

4. Principle

① We suppose that you pull k red balls by the nth iteration. (0≦k≦n)
We consider the probability of pulling a red ball on the n+1th iteration.

② We calculate the sum of further probability from k=0, 1, 2…n. ← **Mutually Exclusive**

③ We calculate p_{n+1}.

5. Method

We distinguish all balls from each other.

・How to determine how many balls there are to take from:

$$\underbrace{(a+b)(a+b+1)(a+b+2)\cdots(a+b+n-1)}_{n}=\frac{(a+b+n-1)!}{(a+b-1)!}$$

・The order of pulling red and white balls:

$${}_nC_k=\frac{n!}{k!\,(n-k)!}$$

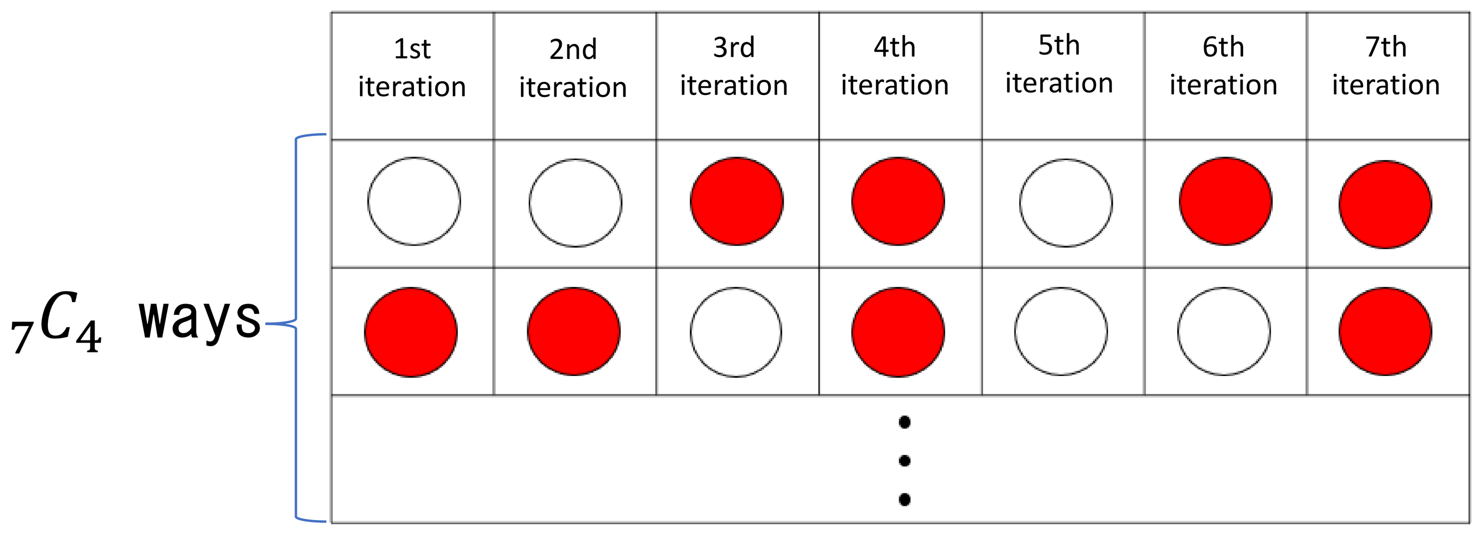
・How to determine how many red balls there are to pull from:

$$\underbrace{a(a+1)(a+2)\cdots(a+k-1)}_{k}=\frac{(a+k-1)!}{(a-1)!}$$

・How to determine how many white balls there are to pull from:

$$\underbrace{b(b+1)(b+2)\cdots(b+n-k-1)}_{n-k}=\frac{(b+n-k-1)!}{(b-1)!}$$

・Example: the case of n=7 and k=4



6. Result

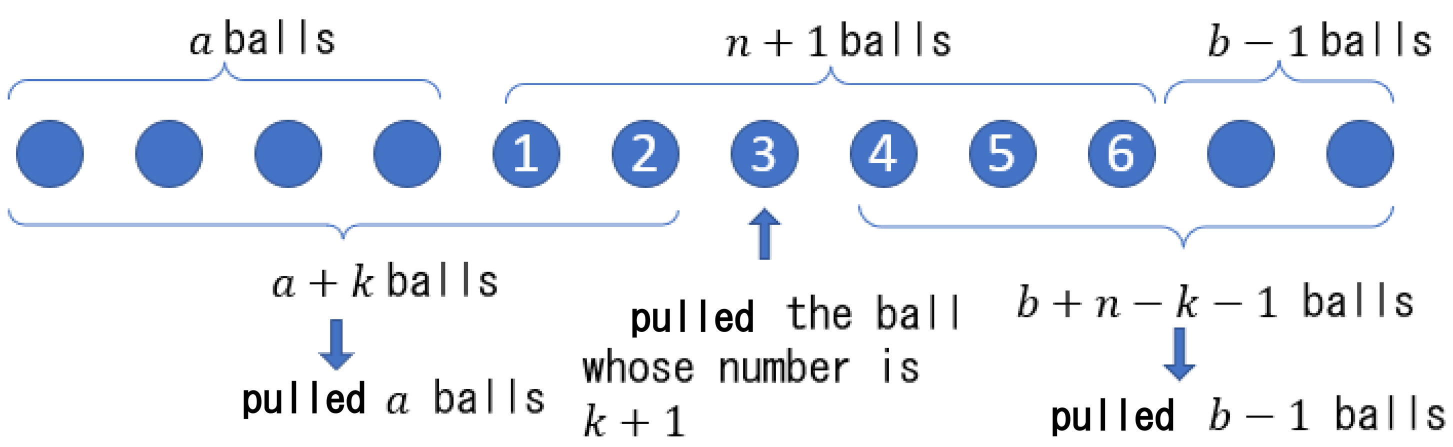
The probability is as follows by using the above way of thinking

$$p_{n+1}=\sum_{k=0}^{n}\frac{(a+k-1)!\cdot(b+n-k-1)!\cdot n!\cdot(a+b-1)!\cdot(a+k)}{(a-1)!\cdot(b-1)!\cdot(n-k)!\cdot k!\cdot(a+b+n-1)!\cdot(a+b+n)}$$

↓ ${}_nC_k=\frac{n!}{k!\,(n-k)!}$

$$p_{n+1}=\sum_{k=0}^{n}\frac{a\cdot{}_{a+k}C_a\cdot{}_{b+n-k-1}C_{b-1}}{(a+b)\cdot{}_{a+b+n}C_{a+b}}$$

$$=\frac{a}{(a+b)\cdot{}_{a+b+n}C_{a+b}}\sum_{k=0}^{n}({}_{a+k}C_a\cdot{}_{b+n-k-1}C_{b-1})$$



The above combination is equal to the combination of pulling a + b from different a + b + n.

$$\sum_{k=0}^{n}{}_{a+k}C_a\cdot{}_{b+n-k-1}C_{b-1}={}_{a+b+n}C_{a+b} \Longrightarrow p_{n+1}=\frac{a}{a+b}$$

7. Reference

A beautiful story of high school math
Probability and Proof of Polya's Pot
URL: https://manabitimes.jp/math/851